1. (16 points) A $k$-uniform hypergraph $G$ is a pair $(X, S)$ where $X$ is a set of vertices and $S \subseteq \binom{X}{k}$ is a set of edges. I.e., each edge in $S$ is a $k$-tuple of vertices. A hypergraph is $c$-colorable if its vertices can be colored with $c$ colors so that no edge is monochromatic (i.e., at least two different colors appear in every edge). Show that if a $k$-uniform hypergraph has less than $2^{k-1}$ edges, then, it is 2-colorable (for any $k \geq 2$).
2. (17 points) In class, for any undirected non-bipartite graph $G(V,E)$, we defined $C_i(G)$ as the expected time to visit each node at least once starting from node $i$, during a random walk. We also defined the cover time $C(G)$ as $\max_{i \in V} C_i(G)$ and showed that $C(G) = O(mn)$, where $n = |V|$ and $m = |E|$. Prove that, independent of the starting node of a random walk, the time taken to visit each node at least once is $\overline{O}(mn \log n)$. 
3. (17 points) Input are two sets $A$ and $B$ with $n$ elements each in the form of arrays. We are also given that $A$ is in sorted order and $B$ may not be in sorted order. In addition, $|A \cap B| = n^{7/12}$. Present an $\tilde{O}(\log n)$ time Las Vegas algorithm to output an element that is common to $A$ and $B$. You can use up to $n^{5/12}$ arbitrary CRCW PRAM processors.
4. (17 points) Input are an $n \times n$ matrix $M$ and an integer $k$. The problem is to compute $M^k$. Show that this can be done in $O(\log n \log k)$ time using $n^3$ CREW PRAM processors.
5. (17 points) Input are two sorted sequences $X = x_1, x_2, \ldots, x_n$ and $Y = y_1, y_2, \ldots, y_n$. Show how to merge these in $O(1)$ time using $n^{1+\epsilon}$ CREW PRAM processors, where $\epsilon$ is any constant $> 0$. 
6. (16 points) Input are two (not necessarily sorted) sequences $X$ and $Y$. Each sequence has $n$ integers in the range $[1, n]$. The problem is to check if there are any common elements between $X$ and $Y$. Present an $O(1)$ time algorithm for this problem. You can employ up to $n$ Common CRCW PRAM processors.