Name: _____

CSE 6512 Randomization in Computing

Exam II; November 30, 2023

Note: You are supposed to give proofs to the time and processor bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (16 points) A k-uniform hypergraph G is a pair (X, S) where X is a set of vertices and $S \subseteq \binom{X}{k}$ is a set of edges. I.e., each edge in S is a k-tuple of vertices. A hypergraph is c-colorable if its vertices can be colored with c colors so that no edge is monochromatic (i.e., at least two different colors appear in every edge). Show that if a k-uniform hypergraph has less than 2^{k-1} edges, then, it is 2-colorable (for any $k \geq 2$).

2. (17 points) In class, for any undirected non-bipartite graph $G(V, E)$, we defined $C_i(G)$ as the expected time to visit each node at least once starting from node i, during a random walk. We also defined the cover time $C(G)$ as $\max_{i \in V} C_i(G)$ and showed that $C(G) = O(mn)$, where $n = |V|$ and $m = |E|$. Prove that, independent of the starting node of a random walk, the time taken to visit each node at least once is $\widetilde{O}(mn \log n)$.

3. (17 points) Input are two sets A and B with n elements each in the form of arrays. We are also given that A is in sorted order and B may not be in sorted order. In addition, $|A \cap B| = n^{7/12}$. Present an $\widetilde{O}(\log n)$ time Las Vegas algorithm to output an element that is common to A and B. You can use up to $n^{5/12}$ arbitrary CRCW PRAM processors.

4. (17 points) Input are an $n \times n$ matrix M and an integer k. The problem is to compute M^k. Show that this can be done in $O(\log n \log k)$ time using n^3 CREW PRAM processors.

5. (17 points) Input are two sorted sequences $X = x_1, x_2, \ldots, x_n$ and $Y = y_1, y_2, \ldots, y_n$. Show how to merge these in $O(1)$ time using $n^{1+\epsilon}$ CREW PRAM processors, where ϵ is any constant > 0.

6. (16 points) Input are two (not necessarily sorted) sequences X and Y. Each sequence has n integers in the range $[1, n]$. The problem is to check if there are any common elements between X and Y. Present an $O(1)$ time algorithm for this problem. You can employ up to n Common CRCW PRAM processors.