CSE 6512 Randomization in Computing

Fall 2023, Homework 2, Due on November 16, 2023

1. *[From MR95, pages 104-105]* Consider the following weighted version of the MAX-SAT problem. Each clause has a positive real weight, and the goal is to maximize the sum of the weights of the satisfied clauses. Show that there is a truth assignment that satisfies clauses the sum of whose weights is at least half of the total clause weight.

2. *[Problem 6.1 from MR95].* Consider a random walk on the infinite line. At each step, the position of the particle is one of the integer points. At the next time step, it moves to one of the two neighboring points equiprobably. Show that the distance of the particle from the origin after n steps is $\widetilde{O}(\sqrt{n \log n})$.

3. Show that we can find the maximum of n elements in $O(1)$ time using $n^{1+\epsilon}$ common CRCW PRAM processors, where ϵ is any constant in the interval $(0, 1)$.

4. The chain sorting problem is defined as follows: The input is a sequence X of n arbitrary elements and the output is the right neighbor of each element of X in sorted order. For example, if $X = 5, 11, 4, 3, 23, 17, 8, 45, 14$, then, the output is $8, 14, 5, 4, 45, 23, 11, \infty, 17$. Show how to solve this problem in $\widetilde{O}(1)$ time using n^2 arbitrary CRCW PRAM processors.

5. Input is a sequence X of n keys where each key is an integer in the range $[1, n^c]$, c being any constant. Show how to sort X in $O(\sqrt{n})$ time using $\sqrt{n}$ CREW PRAM processors.

6. Input are two sets A and B with $|A| = n$, $|B| = m$, and $m < n$. These two sets contain arbitrary real numbers and are not necessarily in sorted order. Present an $\widetilde{O}(\log m)$ time algorithm to compute $A \cap B$. You can use up to n arbitrary CRCW PRAM processors. As an example, if $A = \{8, 12, 3, 6, 11, 15, 4, 55, 32, 18\}$ and $B = \{11, 18, 5, 15, 7, 3\}$, then the elements $18, 11, 3$, and 15 should be output (in any order) in successive cells of the common memory.