Name: ____________________________________

CSE 6512 Randomization in Computing

Exam II (model); Fall 2023

Note: You are supposed to give proofs to the time and processor bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (16 points) Let $G(V,E)$ be a complete graph on n vertices. A *tournament* $T(V,E)$ is nothing but an orientation of the edges of $G(V,E)$. Specifically, for any two vertices u and v in V, exactly one of the directed edges (u,v) and (v,u) is present in $T(V,E)$. A Hamiltonian path in a tournament is a directed path passing through all the vertices. Show that there is a tournament on n vertices that has at least $\frac{n!}{2^{n-1}}$ Hamiltonian paths.

2. (17 points) Input are k sorted sequences $X_1, X_2, \ldots, X_k$ such that $\sum_{i=1}^{k} |X_i| = n$. The problem is to merge these k sequences. Show how to solve this problem in $O(\log k \log\log n)$ time using n CREW PRAM processors.

3. (17 points) Input are k sets represented as (not necessarily sorted) sequences each of length n. Keys in these sequences are integers in the range $[1, n\log^{10} n]$. Show how to compute the intersection of these sets in $\widetilde{O}(\log n \log k)$ time. You can use up to $\frac{nk}{\log n}$ Arbitrary CRCW PRAM processors.

   Extra credit: Show that the above problem can be solved in time $\widetilde{O}\left(\min\left\{\log n \log k, \log n \log\log n + \frac{\log n}{\log\log n}\log k\right\}\right)$ using $\frac{nk}{\log n}$ Arbitrary CRCW PRAM processors.

4. (17 points) Input is an array $A[1:n]$ of arbitrary real numbers. The problem is to find an element of A whose rank in A is in the interval $[\frac{3}{8}n, \frac{5}{8}n]$. Present a Monte Carlo algorithm to solve this problem that runs in $\widetilde{O}(\log n)$ time. You can use upto $n^{1/4}$ CREW PRAM processors. Prove that the output of your algorithm will be correct with high probability.

5. (17 points) Let A and B be two $n \times n$ matrices of arbitrary real numbers. The *tropical product* of A and B is a matrix C such that $C[i,j] = \min_{k=1}^{n} A[i,k] + B[k,j]$, for $1 \le i,j \le n$.

   Present a Las Vegas algorithm that takes as input two $n \times n$ matrices and computes the tropical product of these in $\widetilde{O}(1)$ time. You can use upto n^3 arbitrary CRCW PRAM processors.

6. (16 points) Input are a sequence X of n arbitrary real numbers (not necessarily in sorted order) and another real number r. The problem is to check if there are two elements a and b of X whose sum is r. Present a $\widetilde{O}(\log n)$-time algorithm for this problem. You can use up to n arbitrary CRCW PRAM processors. Assume that the numbers in X are distinct.