Name: ______________________________

CSE 6512 Randomization in Computing

Exam I; March 7, 2016

Note: You are supposed to give proofs to the time bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (25 points) Input is an array A. The array can only be one of two types:

   Type I: A contains n zeros
   Type II: A contains $\frac{n}{2}$ zeros and $\frac{n}{2}$ ones.

   Present an $O(\log n)$ time Monte Carlo algorithm to determine the type of a given array A. Show that the output of your algorithm will be correct with a high probability.

2. (25 points) Input is an array $A[1:n]$ of arbitrary real numbers. This array has $\frac{n}{2}$ copies of one element x and $\frac{n}{4}$ copies of another element y. The other $\frac{n}{4}$ elements are distinct. Present an $O(\log n \log\log n)$ time Monte Carlo algorithm to identify x. Show that the output of your algorithm will be correct with a high probability.

3. (25 points) Input are three $n \times n$ matrices A, B, and C and an integer k. The problem is to check if $A^k B^k = C^k$. Present a Monte Carlo algorithm for this problem that runs in $O(n^2 k \log n)$ time. Show that the output of your algorithm will be correct with high probability.

4. (25 points) Input are two polynomials $f(x)$ and $g(x)$; and two integers m and k. The problem is to check if $[f(x)]^m = [g(x)]^k$. Assume that $m \geq k$. The degree of $f(x)$ is n and the degree of $g(x)$ is d such that $kd = mn$. Present a Monte Carlo algorithm for this problem that runs in time $O(d \log m)$. Prove that the output of your algorithm will be correct with a high probability. Show that this problem can be solved deterministically in $O(mn \log(mn))$ time.