1. (16 points) Input is an array $A[1 : n]$ of real numbers. There are 5 elements in this array that have $\frac{n}{10}$ copies each. The other elements occur exactly once each. The problem is to output any one of these 5 elements. Present an $\widetilde{O}(\log n)$ time Las Vegas algorithm to solve this problem.
2. (17 points) Input is an array $A$ of size $n$. The array can only be of one of the following three types:

- **Type I**: $A$ has all zeros.
- **Type II**: $A$ has all ones.
- **Type III**: $A$ has $\frac{n}{4}$ ones and $\frac{3}{4}n$ zeros.

How will you identify the type of $A$ in $O(\log n)$ time using a Monte Carlo algorithm? Show that the probability of a correct answer will be $\geq 1 - n^{-\alpha}$ for any constant $\alpha \geq 1$. 
3. (16 points) Input is a sequence \( X \) of \( n \) real numbers. The problem is to check if \( X \) is in sorted order. Present a common CRCW PRAM algorithm to solve this problem in \( O(1) \) time. You can use up to \( n \) processors.
4. (17 points) Input are two sorted sequences $X$ and $Y$ of real numbers with $|X| = |Y| = n$. The problem is to merge them. Show how to do this in $O(\log n)$ time using $n$ CREW PRAM processors.
5. (17 points) Input are two sequences $X$ and $Y$ of real numbers in a disk. Each sequence is of length $n$. The problem is to check if they have a common element. Show how to solve this problem in $O\left(\frac{n \log(n/M)}{B \log(M/B)}\right)$ I/O operations. Here $M$ is the main memory size and $B$ is the block size.
6. (17 points) Input is a sequence $X$ of $n$ distinct real numbers residing in a disk. The problem is to output all the elements of $X$ whose ranks in $X$ are in the interval $\left[\frac{n}{2} \pm \frac{M}{2}\right]$. Present an algorithm to solve this problem that makes $O\left(\frac{n}{B}\right)$ I/O operations. Here $M$ is the main memory size and $B$ is the block size. (The rank of an element $k$ in a sequence $X$ of elements is defined as $|\{q \in X : q < k\}| + 1$.)