Name: ______________________________

CSE 3500 Algorithms and Complexity
Exam II, November 15, 2016

Note: You are supposed to give proofs to the time bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (17 points) Input are sequences $X_1, X_2, \ldots, X_m, X_{m+1}$, where $m = 2^{(\log n)^{1/3}}$. For $1 \leq i \leq m$, each X_i is a sorted seuqence of length $\frac{n}{2 \times 2^{(\log n)^{1/3}}}$. X_{m+1} is a sequence of $\frac{n}{2}$ integers in the range $[1, n^{(\log n)^{1/3}}]$ not necessarily in sorted order. The problem is to output a sorted sequence containing all the elements from all of these $(m+1)$ sequences. Present an $O\left(n(\log n)^{1/3}\right)$ time algorithm for this problem.

2. Input are a sequence X of n arbitrary positive real numbers and a positive real number R. The problem is to identify the least key k of X such that: $\sum_{(q \in X : q \leq k)} q \geq R$. For example, if $X = 7, 12, 5, 4, 8, 11, 23, 15, 21, 3$ and $R = 30$ then the answer is $k = 11$ since $\sum_{(q \in X : q \leq 11)} q = 38 > R$. 11 is the least such element of X.

   (a) (12 points) Present an $O(n)$ time algorithm to solve the above problem when X is not in sorted order.

   (b) (6 points) Present an $O(n)$ time algorithm to solve the above problem when X is in sorted order.

3. (16 points) Find a minimum spanning tree for the following graph $G(V, E)$ using either Prim's algorithm or Kruskal's algorithm: $V = \{1, 2, 3, 4, 5, 6, 7\}$ and the edge weights are: $W(1, 2) = 1, W(1, 3) = 2, W(2, 3) = 2, W(2, 4) = 5, W(2, 7) = 4, W(3, 5) = 1, W(3, 6) = 3, W(4, 5) = 3, W(5, 6) = 3, W(5, 7) = 7$, and $W(6, 7) = 4$.

4. (16 points) Use Dijkstra's algorithm to solve the single source shortest paths problem on the following directed graph $G(V, E)$: $V = \{s, 1, 2, 3, 4, 5\}$ and the edge weights are: $W(s,1) = 10, W(s,2) = 2, W(s,5) = 12, W(2,1) = 3, W(2,3) = 2, W(3,2) = 15, W(3,4) = 1, W(3,5) = 2, W(4,3) = 5, W(4,5) = 3, W(5,s) = 4$.

5. (16 points) Solve the following 0/1 knapsack problem using the dynamic programming algorithm discussed in class: $n = 3, m = 8$, $p_1 = 7, w_1 = 2, p_2 = 3, w_2 = 3, p_3 = 2.5$, and $w_3 = 4$.

6. (17 points) Input are strings $S_1, S_2, \ldots, S_n$ from some finite alphabet Σ. The problem is to identify the pair of (distinct) strings for which the edit distance is the minimum (from out of all possible pairs). Present an $O(N^2)$ time algorithm for this problem where $N = \sum_{i=1}^{n} |S_i|$.