Name: ____

CSE 5500 Algorithms; Fall 2018

Exam I; Model

Note: You are supposed to give proofs to the time bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (18 points) Input is an array $A[1:n]$ where there are $\sqrt{n}$ copies of one element and all the other elements are distinct. Present a Las Vegas algorithm to identify the repeated element in $\widetilde{O}(\sqrt{n}\log^2 n)$ time.

2. (17 points) Input is a sequence $X = k_1, k_2, \ldots, k_n$ of arbitrary real numbers. The problem is to find an element q of X such that $\text{rank}(q, X) \in \left[\frac{1}{4}n, \frac{1}{2}n\right]$. Present a Monte Carlo algorithm to find such an element in $O(\log n)$ time. Prove that the output of your algorithm will be correct with high probability.

3. (16 points) $\mathcal{A}$ and $\mathcal{B}$ are two divide-and-conquer recursive algorithms to solve the same problem π. $\mathcal{A}$ partitions π into 36 subproblems of size $\frac{n}{6}$ each and solves each one of them recursively. The time for partitioning and combining the partial solutions is $\Theta(n)$. $\mathcal{B}$ partitions π into $\sqrt{n}$ subproblems each of size $\sqrt{n}$ and solves each one of them recursively. The time for partitioning and combining the partial solutions is n. Which of the two algorithms will you use to solve π? Why?

4. (17 points) Input is a sequence $X = k_1, k_2, \ldots, k_n$ of keys. It is given that each input key is an integer in one of the following ranges: $[a, a+n^{10}], [b, b+n^{20}], [c, c+n^{30}]$, and $[d, d+n^{40}]$, where $a = 1, b = n^{\log n}, c = 2^{\sqrt{n}}$, and $d = 2^n$. Present an $O(n)$ time and $O(n)$ space algorithm to sort X.

5. (17 points) Consider the BFPRT algorithm for selection. In this algorithm we partition the input into groups of size 5 each, find the median of each group, recursively find the median M of these group medians, and use M as the pivot in the quickselect algorithm. We showed in class that this algorithm runs in $O(n)$ time. What happens to the run time of this algorithm if we partition the input into groups of size 3 each (instead of 5)?

6. (17 points) Input are two integer multisets A and B. Recall that a multiset could have many copies of the same element. The problem is to check if A and B are identical, i.e., each integer occurs the same number of times in both sets. Present an $O(n)$ time Monte Carlo algorithm for this problem. Here $n = |A| = |B|$. *Hint: Use fingerprinting.*