Name: ______________________________

CSE 3500 Algorithms and Complexity
Exam I, October 18, 2016

Note: You are supposed to give proofs to the time bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. Prove or disprove:

   - (8 points) If $f(n), g(n), u(n)$, and $v(n)$ are non-negative integer functions of n such that $f(n) = \Theta(g(n))$, and $u(n) = \Theta(v(n))$ then, $f(n) - u(n) = \Theta(g(n) - v(n))$.
   - (8 points) $7n^2 \log n - 15n^{1.5} = \Theta\left(2^{\log n} n \log n\right)$.

2. (17 points) Input is a sequence $X = k_1, k_2, \ldots, k_n$ of arbitrary real numbers. The problem is to identify an element of X that is at least as large as 90% of the elements of X. Present a Monte Carlo algorithm for solving this problem. Your algorithm should run in $O(\log n)$ time. Show that the output of your algorithm will be correct with a high probability.

3. (17 points) Input are a sequence $X = k_1, k_2, \ldots, k_n$ of real numbers (not necessarily in sorted order), an integer l, and a real number s. The problem is to check if there exists an i (where $1 \leq i \leq (n-l+1)$) such that $k_i + k_{i+1} + \cdots + k_{i+l-1} = s$. Present an $O(n)$ time algorithm to solve this problem. Note that l could be a function of n. For example, if $X = 8.5, 5.2, 3.4, 7.1, 11.5, 15.1, 9.3, 2.7$, $l = 3$, and $s = 22.0$, then there exists such an $i = 3$.

4. (16 points) A department has to keep records of its employees such that the following operations can be performed:

   Insert($Name, SSN, salary$): Insert a record for a person whose name is $Name$, social security number is SSN, and salary is $salary$.
   Find_Name(SSN): Return the name of the person whose social security number is SSN;
   Find_SSN($Name$): Return the SSN of the person whose name is $Name$; and
   MaxSalary(): Return the maximum salary of anyone whose record is in the data structure.

   Present a data structure for keeping the records that will take $O(\log n)$ time to perform each of the above operations, n being the number of persons in the department. You can use $O(n)$ space.

5. Solve the following recurrence relations:

(a) (8 points)

$$T(n) = \begin{cases} 1 & n < 5 \\ 125\ T\left(\frac{n}{5}\right) + n^2 & n \geq 5 \end{cases}$$

(b) (9 points)

$$T(n) = \begin{cases} 1 & n < 4 \\ T\left(\frac{n}{16}\right) + T\left(\frac{n}{25}\right) + T\left(\frac{n}{400}\right) + \sqrt{n} & n \geq 4 \end{cases}$$

6. (17 points) Input is a sequence $X = k_1, k_2, \ldots, k_n$ of real numbers not necessarily in sorted order. It is known that each key of X has a value in one of the following disjoint intervals: $[a_1, b_1], [a_2, b_2], \ldots, [a_q, b_q]$. The problem is to compute $n_1, n_2, \ldots, n_q$, where n_i is the number of keys of X that have values in the interval $[a_i, b_i]$, $1 \leq i \leq q$. Present an algorithm for solving this problem that takes $O(n \log q)$ time. The intervals are given and they need not be in sorted order. Also, assume that $q \leq n$.