Name: ____

CSE 6512 Randomization in Computing

Exam I; November 29, 2011

Note: You are supposed to give proofs to the time and processor bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (17 points) Input are two integer multisets A and B. Recall that a multiset could have many copies of the same element. The problem is to check if A and B are identical, i.e., each integer occurs the same number of times in both sets. Present an $O(n)$ time Monte Carlo algorithm for this problem. Here $n = |A| = |B|$. *Hint: Use fingerprinting.*

2. (15 points) $G(V, E)$ is an undirected graph with $|V| = n$. G consists of $\sqrt{n}$ cliques $C_1, C_2, \ldots, C_{\sqrt{n}}$ each of size $\sqrt{n}$. Clique C_i is connected to clique C_{i+1} via a single edge, for $1 \leq i \leq (\sqrt{n} - 1)$. For a random walk on G, show that the cover time is $O(n^2 \log n)$.

3. (16 points) Input is a Boolean formula F on n variables in conjunctive normal form. Let $F = C_1 \wedge C_2 \wedge \cdots \wedge C_m$. Clause C_i has a weight of w_i, for $1 \leq i \leq m$. Show that there exists an assignment to the n variables under which the sum of weights of all the satisfied clauses is $\geq \frac{\sum_{i=1}^{m} w_i}{2}$.

4. (17 points) The chain sorting problem is defined as follows: The input is a sequence X of n arbitrary elements and the output is the right neighbor of each element of X in sorted order. For example, if $X = 5, 11, 4, 3, 23, 17, 8, 45, 14$, then, the output is $8, 14, 5, 4, 45, 23, 11, \infty, 17$. Show how to solve this problem in $\widetilde{O}(1)$ time using n^2 arbitrary CRCW PRAM processors.

5. (17 points) Input is a sequence X of n keys where each key is an integer in the range $[1, n^c]$, c being any constant. Show how to sort X in $O(\sqrt{n})$ time using $\sqrt{n}$ CREW PRAM processors.

6. (18 points) Input are two sets A and B with $|A| = n$, $|B| = m$, and $m < n$. These two sets contain arbitrary real numbers and are not necessarily in sorted order. Present an $\widetilde{O}(\log m)$ time algorithm to compute $A \cap B$. You can use up to n arbitrary CRCW PRAM processors. As an example, if $A = \{8, 12, 3, 6, 11, 15, 4, 55, 32, 18\}$ and $B = \{11, 18, 5, 15, 7, 3\}$, then the elements $18, 11, 3$, and 15 should be output (in any order) in successive cells of the common memory.