CSE 6512, Lecture 17, October 25, 2011

Notes by Mahmoud Maghraby

Theorem (Valiant 1981)

For general keys and deterministic algorithms;

Finding the max of n numbers using n processors needs $\Omega(\log \log n)$ time.

A parallel comparison tree was used by Valiant to prove this theorem. A parallel comparison tree only accounts for the number of comparisons made and hence it is a model that is more powerful than the PRAMs. As a result, the same lower bound readily applies on any of the PRAM models as well.

Fact:

Finding the max of n integers in the range $[1, n^c]$ can be done in $O(1)$ time using *n* Common CRCW PRAM processors, where *c* is any constant.

Fact:

We can find the max of n elements in $O(\log \log n)$ time using $\frac{n}{(\log \log n)}$ Common CRCW PRAM processors.

LEMMA:

We can find the max on n elements in $\tilde{O}(1)$ time using n arbitrary CRCW PRAM processors.

Proof:

Input:

$X = k_1, k_2, k_3, \ldots \ldots \ldots \ldots \ldots .. k_n$

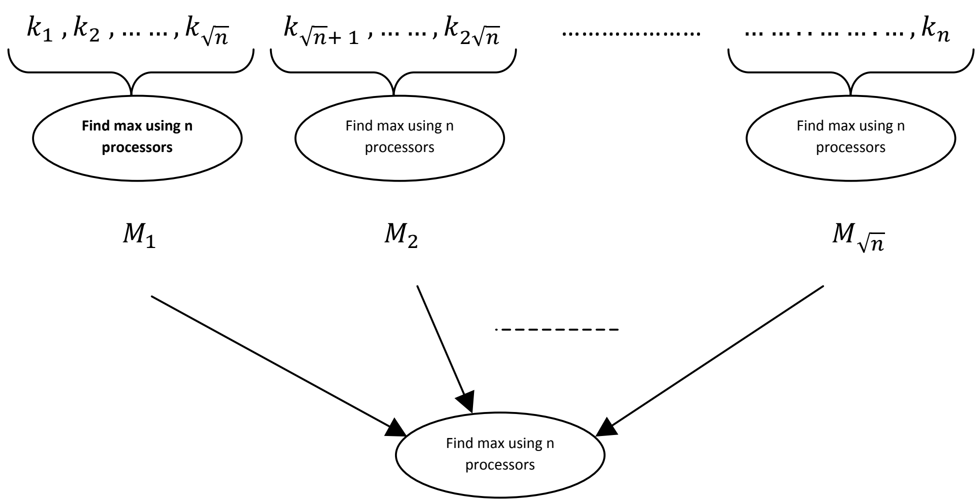
Idea:

1) Pick a random sample S of size $\sqrt{n}$. O(1) Time
2) Find the max M of this sample. O(1) Time
3) for $1 \le i \le n$ in parallel do
   if $k_i < M$ then delete k_i;
   The number of surviving keys is $\tilde{O}(\sqrt{n} \log n) = \tilde{O}(n^{0.51})$
4) Find and output the max of the surviving keys using the following fact.

Fact:

We can find the max of n elements in $O(1)$ time using $n\sqrt{n}$ processors.

Idea:



A problem:

We have to collect the surviving keys and write them in successive cells so that we can proceed with step 4.

We'll place the surviving keys in a region of size $n^{2/3}$ so that each cell in this region will have at most one surviving key.

A Round:

a) Each processor with a surviving key picks a random cell j;
b) The processor reads from j and if j is occupied, it waits for the next round.
c) If the cell j is empty, the processor tries to write its key in j ;
d) The processor reads from j;
e) If j has its key, the processor is done; otherwise, it waits for the next round;

Placement algorithm:

REPEAT

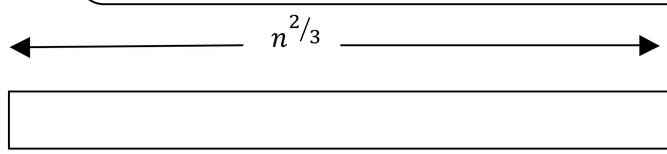
Each processor with a live key participates in a Round.

UNTIL all the keys are placed.

Analysis:

N. Meggido → Parallel Comparison Tree

Rajsekaran and Sen → CRCW PRAM



\# of elements that will be placed = $\tilde{O}(n^{0.51})$

In any given round, the probability that a processor does not succeed is $\leq \frac{n^{0.51}}{n^{2/3}} = O(n^{-0.15})$

$\therefore$ Probability of failure in $c \propto$ successive rounds is $\leq (n^{-0.15\, c \propto})$

R.H.S $\leq n^{-\propto}$ if $c \geq \frac{1}{0.15} = \frac{20}{3}$ □

Prefix Computation

Input:

$k_1, k_2, k_3, \dots, k_n \quad \in \Sigma$

Output:

$K_1,\ K_1 \oplus K_2,\ K_1 \oplus K_2 \oplus K_3, \dots \dots \dots \dots, K_1 \oplus K_2 \dots \dots \dots \oplus K_n$

Where $\oplus$ is any binary, associative, and unit time operation.